

Matrix of Surface Features of Each Task by Category of Cognitive Demand*

Features of Tasks	Memorization		Procedures without Connections				Procedures with Connections					Doing Mathematics				
	F	O	C	E	I	N	D	H	J	L	P	A	B	G	K	M
uses manipulatives			•							•	•		•	•		
uses calculator				•								•			•	
uses a diagram			•					•	•		•		•			•
has "real world" context						•	•					•			•	•
is symbolic/abstract				•	•			•	•	•	•		•	•		
involves multiple steps, actions, or judgments					•	•	•	•	•	•	•	•	•	•	•	•
requires an explanation						•		•				•	•	•	•	•
is "textbook-like"	•	•		•	•	•	•		•			•				

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Vignettes

Scenarios of Six Common Patterns of Task Set Up and Implementation

Scenario A

Mrs. Fox has posted the following problem on the blackboard. As they arrive in the classroom, her students begin to work on the problem immediately with their partners.

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.

- If Ms. Brown's students want their rabbits to have as much room as possible how long would each of the sides of the pen be?
- How long would each of the sides of the pen be if they had only 16 feet of fencing?
- How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Mrs. Fox walks around the room as students begin their work, stopping at different groups to listen in on their conversations and to provide support as needed. She notes that many students started out by describing an assortment of pen configurations that could be built with 24 feet of fencing. As they keep coming up with more configurations, the students begin to realize they needed to keep track of the shapes they have already tried. Some begin to construct a crude table. During this time, Mrs. Fox circulates among the groups asking such questions as, "How do you know you have all the possible pen configurations?", "Which pen has the most room?", and "Do you see a pattern?" These questions lead students to see the need to organize their data, make conjectures, and test them out. As the period draws to a close, none of the groups have completed the task, but most are well on their way to discovering that a square would enclose the greatest amount of area for any given amount of fencing. For homework, Mrs. Fox asks students to summarize what they have learned so far from their exploration and what they want to continue to work on in the next class.

Scenario B

Mr. Chambers has posted the following problem on the blackboard. As they arrive in the classroom, his students begin to work on the problem immediately with their partners.

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.

- If Ms. Brown's students want their rabbits to have as much room as possible how long would each of the sides of the pen be?
- How long would each of the sides of the pen be if they had only 16 feet of fencing?
- How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

As students begin to work on the problem, Mr. Chambers circulates around the room, noticing approvingly that students are taking the task seriously. There are plenty of interesting ideas being discussed although some of them, he had to admit, were not going to lead students toward a strategy for solving the core problem. For example, he hears question such as, "How big are the rabbits?", "How much space does a rabbit need?", and "How much does the fencing cost?" Mr. Chambers decides not to intervene and tell the students how to solve the problem; rather he keeps circulating and observing, hoping that the students will make progress on their own.

With 10 minutes remaining, a few pairs have reached the correct answer for the first question. However, none of the pairs has made progress toward discovering the big mathematical idea: that a square would enclose the greatest amount of area for any given amount of fencing. Mr. Chambers decides to have the students continue to work on the problem for homework and to revisit it again the next day.

Scenario C

Ms. Fagan has posted the following problem on the blackboard. As they arrive in the classroom, her students begin to work on the problem immediately with their partners.

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.

- If Ms. Brown's students want their rabbits to have as much room as possible how long would each of the sides of the pen be?
- How long would each of the sides of the pen be if they had only 16 feet of fencing?
- How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

One-third of the way into a 40-minute period, Ms. Fagan notices that students have made little or no progress. Although many have begun testing out different configurations of pens, most are unsystematic in how they keep track of their work. At this point, Ms. Fagan determines that students will never get to the answer by the end of the period. She puts the following table on the blackboard and tells students they should complete it.

Pen Configuration (length and width)	Length	Width	Perimeter $2(l + w)$	Area $l \times w$
11 x 1				
10 x 2				
9 x 3				
8 x 4				
7 x 5				
6 x 6				
5 x 7				
4 x 8				
3 x 9				
2 x 10				
1 x 11				

Students busily complete the table, relieved that they now know “what to do.” With 5 minutes left in the period, Ms. Fagan asks a pair of students to come to the overhead projector to complete the table and identify which pen configuration would hold the most rabbits. For homework, Ms. Fagan asks the students to construct the same table for 16 feet of fencing.

Scenario D

Ms. Jackson has posted the following problem on the blackboard. As they arrive in the classroom, her students begin to work on the problem immediately with their groups.

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep rabbits.

- If Ms. Brown's students want their rabbits to have as much room as possible how long would each of the sides of the pen be?
- How long would each of the sides of the pen be if they had only 16 feet of fencing?
- How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

As she walks around the room, Ms. Jackson gives each group of students a large sheet of chart paper, explaining that she wants each group to produce a poster showing their work in an organized way. She notices that as soon as they get the chart paper and markers, the students' attention immediately turns to the creation of posters as works of art rather than as the result of mathematical thinking and activity. The students produce elaborate drawings of rabbits and pens and use carefully drawn calligraphy to produce a title. Although some students try to turn the discussion to figuring out the problem, the students who are not engaged in the artistic work are beginning to lose interest and to talk about other things. Ms. Jackson successfully pulls students' attention back to the task when she stops at a group, but the group's attention is not sustained once she leaves.

As the bell rings at the end of the period, Ms. Jackson looks up from the group she currently is talking to and tells the students to drop their posters off at the front of the room. They will discuss the posters in class the following day, she says, as students file out the door.

Scenario E

Mr. Cooper gives his students a worksheet with a set of fraction multiplication problems. The worksheet poses questions such as “Find $\frac{1}{2}$ of $\frac{1}{3}$ using pattern blocks.” For each problem, the worksheet includes duplicate diagrams of two hexagons pushed together to represent the unit whole. Under the first two hexagons, students are asked to use pattern blocks to **show $\frac{1}{3}$** (for the problem “Find $\frac{1}{2}$ of $\frac{1}{3}$ ”); under the second two hexagons, students are asked to **show $\frac{1}{2}$ of $\frac{1}{3}$** (for the same problem).

Mr. Cooper begins the lesson by having a successful student model how to solve $\frac{1}{6}$ of $\frac{1}{2}$. As students set out to work on their own, Mr. Cooper emphasizes the importance of using the sketches and the blocks to keep track of how they are thinking through the problems. As he moves around the room, he watches and listens and reminds students to work with their partners. He also pairs students with conflicting sketches in order for them to help one another decide what is right. After some time, a pair of students presents their solution to one of the problems. There is a discussion of what is meant by the “unit whole,” during which students generate real-world examples. Mr. Cooper asks students to finish the problems for homework and conducts a closing discussion, looking across the results of the first three problems. They reflect on the relative sizes of products and factors, noticing that they are multiplying but the products are smaller than what they started with (this is different than multiplication with whole numbers). Mr. Cooper adds an additional homework question that asks student to explain why multiplying a number by a fraction generates a smaller number than the original number.

Scenario F

Ms. Gorman gives her students a worksheet with a set of fraction multiplication problems. The worksheet poses questions such as “Find $\frac{1}{2}$ of $\frac{1}{3}$ using pattern blocks.” For each problem, the worksheet includes duplicate diagrams of two hexagons pushed together to represent the unit whole. Under the first two hexagons, students are asked to use pattern blocks to **show $\frac{1}{3}$** (for the problem “Find $\frac{1}{2}$ of $\frac{1}{3}$ ”); under the second two hexagons, students are asked to **show $\frac{1}{2}$ of $\frac{1}{3}$** (for the same problem).

Ms. Gorman starts by asking a successful student to show how to use pattern blocks to find $\frac{1}{6}$ of $\frac{1}{2}$. The students do not have any questions after the presentation, so Ms. Gorman lets the students start working on the problem set with a partner. Ms. Gorman visits each pair making sure they are “doing it correctly,” sometimes helping them move the patterns blocks into place. Some students have difficulty with the unit whole comprised of two hexagons, rather than one. Ms. Gorman tries to make sure the students learn the right steps for using the blocks to solve at least one example so they can use the same procedure for other problems. Some students can get through the procedure if guided directly by Ms. Gorman, but she is not sure they really understand what they are doing. Most students do not complete more than one or two problems.

Answers:

Scenario	Pattern
A	Doing Mathematics Maintained
B	Doing Mathematics Into Unsystematic Exploration
C	Doing Mathematics Into Procedures Without Connections
D	Doing Mathematics Into No Mathematical Activity
E	Procedures With Connections Maintained
F	Procedures With Connections Into Procedures Without Connections

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

1

TASK A

Manipulatives or Tools Available: Calculator

Treena won a 7-day scholarship worth \$1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are \$335 by air or \$125 by train. At the camp she must choose between a week of individual instruction at \$60 per day or a week of group instruction at \$40 per day. Treena's food and other expenses are fixed at \$45 per day. If she does not plan to spend any money other than the scholarship, what are *all* choices of travel and instruction plans she could afford to make? Explain which option you think Treena should select and why.

Source: Kenney and Silver 1997, p.108

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

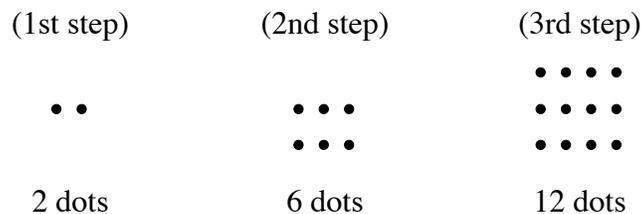
2

TASK B

Manipulatives or Tools Available: Counters

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show *all* your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.



Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots.

Explain how Marcy could do this *and* give the answer that she should get for the number of dots in the 20th step.

Source: Kenney and Silver 1997, p.240

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

TASK C

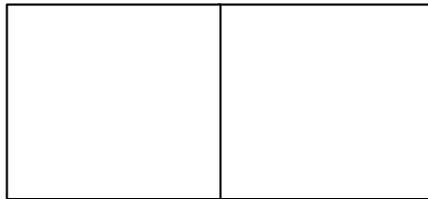
Manipulatives or Tools Available: Square Pattern Tiles

Using the side of a square pattern tile as a measure, find the perimeter (i.e., distance around) of each train in the pattern block figure shown below.

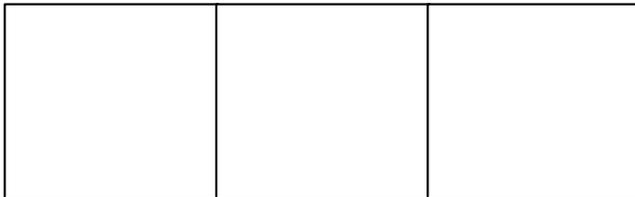
Train 1



Train 2



Train 3



Middle School Sort*

Tasks for Use in Principals' Sorting Activity

4

TASK D

Manipulatives or Tools Available: None

Part A: The place kicker on the North High School football team has made 13 out of 20 field goals so far this season. The place kicker on the South High School football team has made 15 out of 25 field goals so far this season. Which player has made the greatest percentage of field goals?

Part B: If the "better" player does not play for the rest of the season, how many field goals would the other player need to make in order take the lead in terms of greatest percentage of field goals?

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

5

TASK E

Manipulatives or Tools Available: Calculator

Divide using paper and pencil. Check your answer with a calculator and round the decimal to the nearest thousandth.

$$\frac{525}{1.3}$$

$$\frac{52.75}{7.25}$$

$$\frac{30.459}{.12}$$

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

6

TASK F

Manipulatives or Tools Available: None

Match the property name with the appropriate equation.

1. Commutative property of addition	a. $r(s+t) = rs + rt$
2. Commutative property of multiplication	b. $x \times 1/x = 1$
3. Associative property of addition	c. $-y + x = x + (-y)$
4. Associative property of multiplication	d. $a/b + -a/b = 0$
5. Identity property of addition	e. $y \times (zx) = (y z) \times x$
6. Identity property of multiplication	f. $1 \times (xy) = xy$
7. Inverse property of addition	g. $d \times 0 = 0$ and $0 \times d = 0$
8. Inverse property of multiplication	h. $x + (b + c) = (x + b) + c$
9. Distributive property	i. $y + o = y$
10. Property of zero for multiplication	j. $p \times q = q \times p$

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

7

TASK G

Manipulatives or Tools Available: Base-10 Blocks, grid paper

.08 .8 .080 .008000

Make three observations about the relative size of the above 4 numbers. Be sure to explain your observations as clearly as possible. Feel free to illustrate your observations if you feel it would help others understand them.

Adapted from QUASAR Project: QUASAR Cognitive Assessment Instrument-Release Task

Middle School Sort*

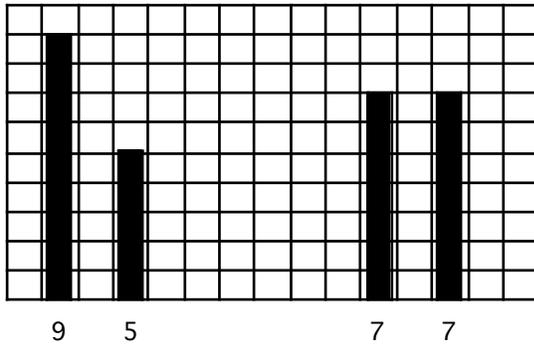
Tasks for Use in Principals' Sorting Activity

8

TASK H

Manipulatives or Tools Available: Grid Paper

The pairs of numbers in a–d below represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.



a) 14 and 8 b) 16 and 7 c) 7 and 12 d) 13 and 15

By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total # of cubes is now distributed into 2 columns of equal height. And that is what average means.

Taken from *Visual Mathematics Course I*, The Math Learning Center, 1995, Lesson 10, Follow-up Student Activity 10.1, #1, p. 121.

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

9

TASK I

Manipulatives or Tools Available: None

Write and solve a proportion for each.

17 is what percent of 68?

What is 15% of 60?

8 is 10% of what number?

24 is 25% of what number?

28 is what percent of 140?

What is 60% of 45?

36 is what percent of 90.

What is 80% of 120?

21 is 30% of what number?

Middle School Sort*

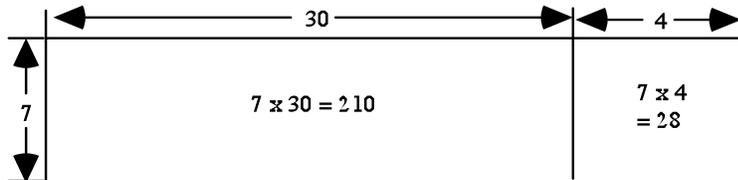
Tasks for Use in Principals' Sorting Activity

10

TASK J

Manipulatives or Tools Available: None

One method of mentally computing 7×34 is illustrated in the diagram below:



Mentally compute these products. Then sketch a diagram that describes your methods for each.

- a) 27×3
- b) 325×4

Adapted from *Visual Mathematics Course I*, The Math Learning Center, 1995, Lesson 37, Focus Teacher Activity, pp. 457–63.

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

11

TASK K

Manipulatives or Tools Available: Calculator with scientific functions

Penny's mother told her that several of her great-great-great-grandparents fought in the Civil War. Penny thought this was interesting, and she wondered how many great-great-great-grandparents she actually had. When she found that number, she wondered how many generations back she'd have to go until she could count over 100 ancestral grandparents---or 1,000, or 10,000, or even 100,000. When she found out, she was amazed and also pretty glad she had a calculator. How do you think Penny might have figured out all of this information? Explain and justify your method as clearly and completely as possible.

Adapted from Blackline Master 3B from *AW Mathematics: Mathematics with Calculators: Resources for Teachers, Grade 6*, Section 3, Activity 2. Addison-Wesley Publishing Company, 1988. Copyright 1988, Addison-Wesley Publishing Co., Inc.

Middle School Sort*
Tasks for Use in Principals' Sorting Activity

12

TASK L

Manipulatives or Tools Available: Base-10 Blocks

Using Base-10 blocks, show that 0.292 is less than 0.3.

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

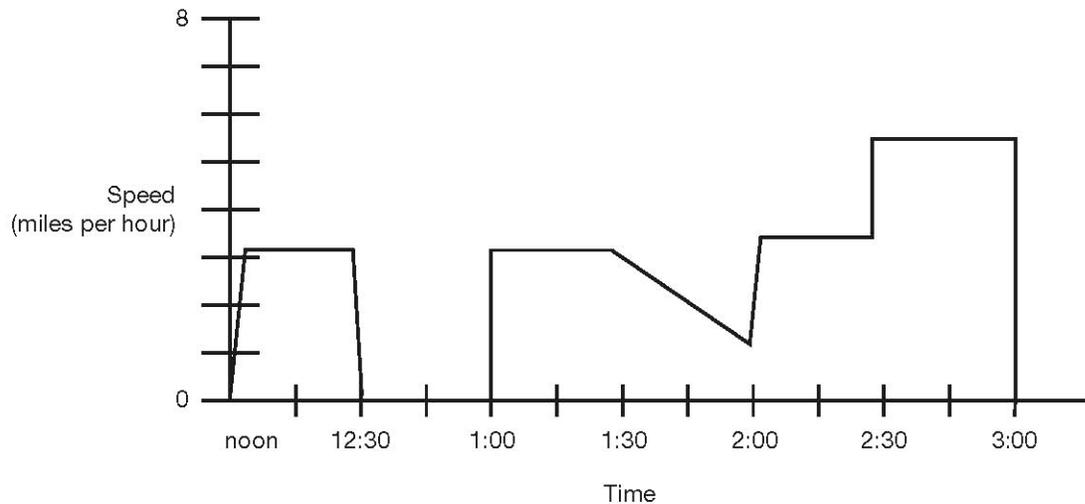
13

TASK M

Manipulatives or Tools Available: None

Use the following information and the graph to write a story about Tony's walk.

At noon, Tony started walking to his grandmother's house. He arrived at her house at 3:00 p.m. The graph below shows Tony's speed in miles per hour throughout his walk.



Write a story about Tony's walk. In your story, describe what Tony might have been doing at the different times.

Taken from the *QUASAR Project: QUASAR Cognitive Assessment Instrument-Release Task*

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

14

TASK N

Manipulatives/Tools: None

The cost of a sweater at J. C. Penney's was \$45.00. At the "Day and Night Sale" it was marked 30% off of the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.

Middle School Sort*

Tasks for Use in Principals' Sorting Activity

15

TASK O

Manipulatives or Tools Available: None

Give the fraction and percent for each decimal.

$$.20 = \underline{\quad} = \underline{\quad}$$

$$.25 = \underline{\quad} = \underline{\quad}$$

$$.33 = \underline{\quad} = \underline{\quad}$$

$$.50 = \underline{\quad} = \underline{\quad}$$

$$.66 = \underline{\quad} = \underline{\quad}$$

$$.75 = \underline{\quad} = \underline{\quad}$$

Middle School Sort*

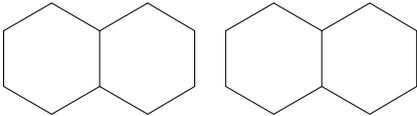
Tasks for Use in Principals' Sorting Activity

16

TASK P

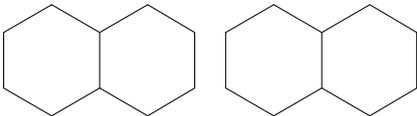
Manipulatives or Tools Available: Pattern Blocks

Find $\frac{1}{2}$ of $\frac{1}{3}$. Use pattern blocks. Draw your answer.



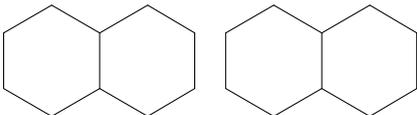
$\frac{1}{2}$ of $\frac{1}{3}$ or $\frac{1}{2} \times \frac{1}{3} =$

Find $\frac{1}{3}$ of $\frac{1}{4}$. Use pattern blocks. Draw your answer.



$\frac{1}{3}$ of $\frac{1}{4}$ or $\frac{1}{3} \times \frac{1}{4} =$

Find $\frac{1}{4}$ of $\frac{1}{3}$. Use pattern blocks. Draw your answer.



$\frac{1}{4}$ of $\frac{1}{3}$ or $\frac{1}{4} \times \frac{1}{3} =$